(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199320

Roll No.

B. Tech.

(SEM. III) (ODD SEM.) THEORY EXAMINATION, 2014-15

MATHEMATICS-III

Time : **3** Hours]   [Total Marks : **100**

Note : Attempt All Questions. All Questions carry equal marks

1   Attempt any four parts of the following :   5×4=20

   (a)   State Cauchy-Riemann theorem for an analytic function. Test the analyticity of the following Function :

$$f(z) = \frac{\left(x^3 - y^3\right) + i\left(x^3 + y^3\right)}{x^2 + y^2}, \text{ if } z \neq 0 \text{ and}$$

          $= 0$ if $z = 0$

   (b)   State Cauchy- integral theorem for an analytic function. Verify this theorem by integrating the function $z^3 + iz$ along the boundary of the rectangle with vertices $+ 1, -1, i, -i$.

   (c)   Show that the function $u = \frac{1}{2}\log\left(x^2 + y^2\right)$ is harmonic.

       Find the harmonic conjugate of u.

(d) Evaluate the integral $\int \dfrac{e^{2z}}{(z+1)^5} dZ$, around the boundary

of the circle $|z| = 2$.

(e) Find the Taylor series expansion of the function $\tan^{-1} z$ about the point $z = \pi/4$.

(f) Evaluate the integral $\int_0^\pi \dfrac{\cos^2 3\theta}{5 - 4\cos 2\theta} d\theta$.

2 Attempt any two parts of the following :   10×2=20

(a) Find the Fourier transform of the following function
$f(x) = 1-x^2$, if $|x| \leq 1$ and $f(x) = 0$, if $|x| > 1$

(b) Using Z — transform solve the following difference equation

$Y_{n+2} - (2\cos\alpha) Y_{n+1} + Y_n = 7^n$ with the conditions that $Y_0 = 5$, $Y_1 = 1$.

(c) State the convolution theorem for Fourier transform. Prove that the Fourier transform of the convolution of the two functions equal to the product of their Fourier transforms.

3 Attempt any two parts of the following :   10×2=20

(a) Define skewness and kurtosis of a distribution. The first four moments of a distribution are 0, 2.5, 0.7, and 18.71. Find the coefficient of skewness and kurtosis.

(b) Fit a second degree parabola to the following data :

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	8	13	5

(c) Define coefficient of correlation and regression. If θ is the acute angle between the two lines of regression then prove that

$$\tan\theta = \frac{1-r^2}{r} \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

where r, σ_x, σ_y have their usual meanings. Give the significance of the formula when r = 0 and r = ± 1.

4 Attempt any two parts of the following :   10×2=20

(a) Derive Newton – Raphson's method to find a root of the equation f(x) = 0. Prove that this method has quadratic convergence.

(b) Apply Newton's divided difference method to obtain an interpolatory polynomial for the following data.

x	3	5	7	9	11	13
$f(x)$	31	51	17	19	90	110

(c) Obtain Lagrange's Interpolatory for the following data :

x	1	3	5	7	10
$f(x)$	13	31	25	37	101

Find the values of f(4) and f(8.5).

5 Attempt any two parts of the following : **10×2=20**

(a) Solve the following system of linear equations using Gauss-Seidel method

$10x + 3y + 7z = 41$,

$3x + 20y + 17z = 101$,

$x + 19y + 23z = 201$,

perform three iterations.

(b) State Simpson's three-eighth rule. Using this rule evaluate the following integral

$$\int_0^6 \frac{x}{1+x^5} dx$$

(c) State Runge-Kutta method of fourth order. Using this method find the values of y (0.2), y(0.4) and y(0.6) for the following initial value problem

$\frac{dy}{dx} = x^3 - y^3$ with condition that y(0) = 1

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